Dynamical Manifestation of the Goldstone Phenomenon at 1-loop

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Abstract

We have calculated the damping rate $\Gamma(|\mathbf{k}|)$ for classical on-shell Goldstone modes of the $O(2)$ symmetric scalar fields propagating in a thermal medium of the broken symmetry phase taking into account the effect of the explicit symmetry breaking. The result of the one-loop analysis can be expanded around $\Gamma(0)$, which depends non-analytically on the parameter of the explicit symmetry breaking, h. $\Gamma(0)$ vanishes when $h \to 0$, demonstrating in this way the absence of the restoring force, when the equilibrium direction of the symmetry breaking is modulated homogeneously.

The equivalence of the vacuum states in a system with spontaneous symmetry breaking should manifest itself also in the dynamical evolution of fluctuations and external signals: when an external signal superimposed on the actual vacuum realizes the transition over to an equivalent vacuum state no relaxation to the initial vacuum should occur. In this note we present an explicit analysis of the near equilibrium, linear relaxation phenomenon in an $O(2)$-symmetric scalar field theory from this point of view. It is convenient (and also more realistic) to study the problem in a system with additional explicit breaking of the symmetry, and follow what happens when its strength

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goes to zero. We are going to derive an explicit formula for the decay rate of a homogeneous Goldstone modulation of the vacuum displaying the expected behaviour in a rather non-trivial way.

Our investigation is an extension of our recent detailed analysis of the one-loop finite temperature dynamics of the $O(N)$-symmetric scalar field theories [1]. (See also [2, 3].) The detailed formalism presented there permits us to restrict the presentation to the most important relations, since the formal details are very similar to the case with zero explicit symmetry breaking. We have specialized the discussion of the *linear response function* to the $O(2)$ case just to simplify some formulae. The conclusions appear to be of intrinsic validity for the dynamical effect of the Goldstone phenomenon.

The study of the Goldstone-propagation through thermal medium is of particular interest for the interpretation of the pion-sigma dynamics in heavy ion collisions. Several recent field theoretical studies have dealt with this problem [4, 5, 6] mainly concentrating on the evolution of the heavy (sigma or Higgs) component. Though it has been observed in [5] that homogeneous pion condensates do not decay, the origins of this statement have not been fully explored. The effect of explicit symmetry breaking was taken into account by some authors by treating the pions as effectively massive degrees of freedom, without exploiting the consequences of the approximate symmetry [7, 8].

The Lagrangian of the $O(2)$ symmetric scalar field theory with explicit symmetry breaking is the following:

$$L=\frac{1}{2}\left(\partial_\mu\phi_1\right)^2+\frac{1}{2}\left(\partial_\mu\phi_2\right)^2-\frac{1}{2}\mu^2\left(\phi_1^2+\phi_2^2\right)-\frac{\lambda}{24}\left(\phi_1^2+\phi_2^2\right)^2+h\phi_1. \tag{1}$$

To account for the symmetry breaking one shifts the Φ_1-field:

$$\phi_1\to\bar{\Phi}+\phi_1, \tag{2}$$

and we assume that $\bar{\Phi}$ is given by its equilibrium value. Then we are led to the following shifted Lagrangian:

$$\begin{aligned} L &= \frac{1}{2}\left(\partial_\mu\phi_1\right)^2+\frac{1}{2}(\partial_\mu\phi_2)^2-\frac{1}{2}m_1^2\phi_1^2-\frac{1}{2}m_2^2\phi_2^2+\phi_1\left(h-\mu^2\bar{\Phi}-\frac{\lambda}{6}\bar{\Phi}^3\right) \\ &- \frac{\lambda}{24}\left(4\bar{\Phi}\phi_1^3+\phi_1^4+4\bar{\Phi}\phi_1\phi_2^2+2\phi_1^2\phi_2^2+\phi_2^4\right) \end{aligned} \tag{3}$$

with the notations

$$m_1^2 = \mu^2 + \frac{\lambda}{2}\bar{\Phi}^2, \qquad m_2^2 = \mu^2 + \frac{\lambda}{6}\bar{\Phi}^2. \tag{4}$$

Next we sketch the steps followed in the derivation of the effective equations for the propagation of a non-thermal signal on a thermalised background:

1. Decomposition of the fields ϕ_i into the sum of high-frequency, thermalised (φ_i) and low-frequency, non-thermal (Φ_i) components;

2. Derivation of the equations of motion for Φ_i in the background of the thermal components;

3. Averaging the equations over the thermal background, retaining only contributions from the two-point functions of the thermalised fields;

4. Approximating the two-point functions resulting from step 3 by expressions with at most linear functional dependence on the non-thermal fields, what is sufficient for the calculation of the linear response function of the theory:

$$\begin{aligned}\langle\varphi_i(x)\varphi_j(y)\rangle \approx \langle\varphi_i(x)\varphi_j(y)\rangle|_{\Phi=0} + \int dz\, \left.\frac{\delta\langle\varphi_i(x)\varphi_j(y)\rangle}{\delta\Phi_l(z)}\right|_{\Phi=0} \cdot \Phi_l(z) \equiv \\ \langle\varphi_i(x)\varphi_j(y)\rangle^{(0)} + \langle\varphi_i(x)\varphi_j(y)\rangle^{(1)}.\end{aligned} \tag{5}$$

The first (Φ_i-independent) term is different from zero only for $i = j$ and it is given by its equilibrium expression, with thermal population restricted to the $p_0 > \Lambda$ range:

$$\begin{aligned}\langle\varphi_i\varphi_j\rangle^{(0)} = \delta_{ij}\int\frac{d^4p}{(2\pi)^4}2\pi\delta(p^2 - M_i^2)(\Theta(p_0) + \tilde{n}(|p_0|)), \\ \tilde{n}(x) = \Theta(x - \Lambda)\tfrac{1}{e^{\beta x}-1}.\end{aligned} \tag{6}$$

In the equilibrium distribution one makes use of the physical masses of the fields, to be deduced below (see Eq.(13)).

The linearised equations of motion resulting from the above described procedure are the following:

$$\begin{aligned}\left(\partial^2 + m_1^2 + \frac{\lambda}{2}\langle\varphi_1^2\rangle^{(0)} + \frac{\lambda}{6}\langle\varphi_2^2\rangle^{(0)}\right)\Phi_1 &= -\frac{\lambda}{2}\bar{\Phi}\langle\varphi_1^2\rangle^{(1)} - \frac{\lambda}{6}\bar{\Phi}\langle\varphi_2^2\rangle^{(1)}, \\ \left(\partial^2 + m_2^2 + \frac{\lambda}{2}\langle\varphi_2^2\rangle^{(0)} + \frac{\lambda}{6}\langle\varphi_1^2\rangle^{(0)}\right)\Phi_2 &= -\frac{\lambda}{3}\bar{\Phi}\langle\varphi_1\varphi_2\rangle^{(1)}.\end{aligned} \tag{7}$$

In addition the equation for the equilibrium (static) value of the vacuum expectation, $\bar{\Phi}$ is also found:

$$m_2^2\bar{\Phi} - h + \frac{\lambda}{2}\bar{\Phi}\langle\varphi_1^2\rangle^{(0)} + \frac{\lambda}{6}\bar{\Phi}\langle\varphi_2^2\rangle^{(0)} = 0. \tag{8}$$

Below for the sake of simpler notation, we shall use the following conventional abbreviation:

$$\langle\varphi_i(x)\varphi_j(y)\rangle \equiv \Delta_{ij}(x,y). \tag{9}$$

In order to proceed with the discussion of the decay rate of the Goldstone condensate we have to derive an equation for $\langle\varphi_1\varphi_2\rangle^{(1)}$, which determines the propagation of Φ_2:

$$\left(\partial_x^2 - \partial_y^2 + \frac{\lambda}{3}\bar{\Phi}^2\right)\Delta_{12}^{(1)}(x,y) = -\frac{\lambda}{3}\bar{\Phi}\left(\Phi_2(x)\Delta_{22}^{(0)}(x-y) - \Phi_2(y)\Delta_{11}^{(0)}(x-y)\right). \tag{10}$$

After performing the combined Wigner-Fourier transformation [9, 10, 1] of this equation one has

$$\left(2p\cdot k + \frac{\lambda}{3}\bar{\Phi}^2\right)\Delta_{12}^{(1)}(k,p) = -\frac{\lambda}{3}\bar{\Phi}\Phi_2(k)\left(\Delta_{22}^{(0)}(p+k/2) - \Delta_{11}^{(0)}(p-k/2)\right). \tag{11}$$

Finally, the effective linear wave equation for the field Φ_2 looks like:

$$\begin{aligned}\left(-k^2 + \tfrac{h}{\bar{\Phi}}\right)\Phi_2(k) &= \tfrac{\lambda}{3}\int\frac{d^4p}{(2\pi)^4}\left(\Delta_{11}^{(0)}(p) - \Delta_{22}^{(0)}(p)\right)\Phi_2(k)\\ &+ \left(\tfrac{\lambda}{3}\bar{\Phi}\right)^2\Phi_2(k)\int\frac{d^4p}{(2\pi)^4}\frac{1}{2p\cdot k + \frac{\lambda}{3}\bar{\Phi}^2}\left(\Delta_{22}^{(0)}(p+k/2) - \Delta_{11}^{(0)}(p-k/2)\right)\end{aligned} \tag{12}$$

For $k = 0$ the right hand side of this equation vanishes, what reflects the effect of the Goldstone-theorem on the self-energy function of Φ_2. A similar analysis of the first equation of motion in Eq.(7) shows that the diagonal thermal two-point functions appearing on the right hand side do not contribute any local term, which could be interpreted as the correction of the mass-square appearing on the left hand side. Therefore

$$M_H^2 = \frac{\lambda}{3}\bar{\Phi}^2 + \frac{h}{\bar{\Phi}}, \qquad M_G^2 = \frac{h}{\bar{\Phi}}. \tag{13}$$

The decay rate of the Φ_2-field is determined by the imaginary part of the self-energy function appearing on the right hand side of (12):

$$(\frac{\lambda}{3}\bar{\Phi})^2\mathrm{Im}\left[\int\frac{d^4p}{(2\pi)^4}\frac{1}{2p\cdot k+M_H^2-M_G^2}\left(\Delta_{22}^{(0)}(p+k/2)-\Delta_{11}^{(0)}(p-k/2)\right)\right]. \tag{14}$$

The kinematical analysis of the range of variables contributing to this integral is much more involved than in the case when the symmetry breaking happens exclusively spontaneously (see [7] for a detailed discussion). After the straightforward but tedious procedure we find for the imaginary part of the self-energy function:

$$\begin{aligned}
&\mathrm{Im}\ \Pi_2(k_0,|\mathbf{k}|) = \left(\frac{\lambda}{3}\bar{\Phi}\right)^2\frac{1}{16\pi|\mathbf{k}|}\times \\
&\left[\Theta(-k^2)\int_{\alpha_-}^{\infty}ds\ \ (n(s)-n(s-k_0))+\Theta(-k^2)\int_{\alpha_+}^{\infty}ds\ \ (n(s+k_0)-n(s))\right. \\
+&\ \ \Theta(k^2)\Theta(M_G^2-M_H^2)\Theta\left((M_H-M_G)^2-k^2\right)\int_{\alpha_-}^{\alpha_+}ds\ (n(s)-n(s-k_0)) \\
+&\ \ \Theta(k^2)\Theta(M_H^2-M_G^2)\Theta\left((M_G-M_H)^2-k^2\right)\int_{\alpha_+}^{\alpha_-}ds\ \ (n(s+k_0)-n(s)) \\
-&\ \ \left.\Theta(k^2)\Theta\left(k^2-(M_H+M_G)^2\right)\int_{\alpha_-}^{\alpha_+}ds\ \ (1+n(s)+n(k_0-s))\right],
\end{aligned} \tag{15}$$

with

$$\alpha_\pm = \left|\frac{1}{2k^2}\left(k_0\left(k^2-M_H^2+M_G^2\right)\pm|\mathbf{k}|\sqrt{(k^2-M_H^2+M_G^2)^2-4k^2M_G^2}\right)\right|. \tag{16}$$

In case of on-shell propagation ($k_0^2-|\mathbf{k}|^2=M_G^2$) and $M_G<M_H$, only the fourth term contributes. With its help for the decay rate one finds

$$\Gamma = \frac{(\lambda\bar{\Phi}/3)^2\Theta(M_H-2M_G)}{32\pi|\mathbf{k}|\sqrt{|\mathbf{k}|^2+M_G^2}}\left[\int_{\alpha_+}^{\alpha_++k_0}ds\ n(s)-\int_{\alpha_-}^{\alpha_-+k_0}ds\ n(s)\right]. \tag{17}$$

For $M_G = 0$ and $k_0 \neq 0$ it reproduces our recent result for the damping rate of a Goldstone wave without explicit symmetry breaking [1]:

$$\Gamma(\mathbf{k})|_{M_G=0} = \left(\frac{\lambda}{3}\bar{\Phi}\right)^2 \frac{1}{32\pi|\mathbf{k}|}\, n\left(\frac{M_H^2}{4|\mathbf{k}|}\right)). \tag{18}$$

For $M_G \neq 0$, the damping rate is a continuous function of $|\mathbf{k}|^2$ and can be expanded around $|\mathbf{k}| = 0$:

$$\begin{aligned}\Gamma(|\mathbf{k}| = 0) = \; & \frac{\lambda^2\bar{\Phi}^2}{288\pi}\Theta\left(M_H - 2M_G\right)\frac{M_H}{M_G^3}\sqrt{M_H^2 - 4M_G^2}\times \\ & \left(\exp\left(-\frac{\beta M_H^2}{2M_G}\right) - \exp\left(-\frac{\beta(M_H^2 + M_G^2)}{2M_G}\right)\right)\times \\ & \left(\exp\left(-\frac{\beta(M_G^2 + M_H^2)}{2M_G}\right) - 1\right)^{-1}\left(\exp\left(-\frac{\beta M_H^2}{2M_G}\right) - 1\right)^{-1}.\end{aligned} \tag{19}$$

In particular, it can be evaluated for $T \gg M_G, M_H$, with a result which exactly coincides with the classical expression for the decay rate. This latter arises from Eq.(17) with the replacement $n(s) \to n_{cl}(s) = T/s$. It can be derived from the classical evolution equations of the $O(2)$ symmetric scalar model as explained in [11].

However, for $M_G \to 0$ the expression (19) does not follow the classical theory, but vanishes non-analytically as $\exp(-\beta M_H^2/2M_G)$. This result demonstrates that not only the self-energy function, but *also the decay rate vanishes for* $|\mathbf{k}| = 0$*, when the strength of the explicit symmetry breaking goes to zero.*

The quantity $\Gamma(h, |\mathbf{k}|)$ arises at this order of the perturbation theory as the difference of the rates of two processes: the transformation of the Goldstone-wave into a Higgs-wave with the absorption of a (high-frequency) thermal Goldstone-particle and its inverse. From the macroscopic point of view our result presents the rate of transformation of a Goldstone-signal into Higgs-signal when propagating through the thermalised medium. In the approximation when one assumes that a single act of transformation is not followed by any further interaction with the thermal bath, it gives also the inverse life-time of the Goldstone-wave. However, if one anticipates further multiple transformations between the Goldstone and the Higgs forms

of propagation, then the damping rate of the original Goldstone signal will considerably increase for small, but finite h due to the more efficient damping of the $k_0 \approx T$ Higgs-waves. However, for $h = 0$ the full rate still vanishes, since the Goldstone-to-Higgs transformation rate itself is zero. A recent analysis of A. Jakovác [12] presents a systematic way for taking into account the effect of the larger Higgs-width in the Goldstone-propagator.

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